NAG C Library Function Document

nag_dpbrfs (f07hhc)

1 Purpose

nag_dpbrfs (f07hhc) returns error bounds for the solution of a real symmetric positive-definite band system of linear equations with multiple right-hand sides, $AX = B$. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```
void nag_dpbrfs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kd,
     Integer nrhs, const double ab[], Integer pdab, const double afb[],
     Integer pdafb, const double b[], Integer pdb, double x[], Integer pdx,
     double ferr[], double berr[], NagError *fail)
```

3 Description

nag_dpbrfs (f07hhc) returns the backward errors and estimated bounds on the forward errors for the solution of a real symmetric positive-definite band system of linear equations with multiple right-hand sides $AX = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of nag_dpbrfs (f07hhc) in terms of a single right-hand side b and solution x.

Given a computed solution x, the function computes the *component-wise backward error* β. This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$
$$|\delta a_{ij}| \le \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \le \beta |b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, the f07 Chapter Introduction.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A has been factorized, as follows:

if **uplo** $=$ **Nag_Upper**, the upper triangular part of A is stored and A is factorized as U^TU, where U is upper triangular;

if **uplo** $=$ **Nag_Lower**, the lower triangular part of A is stored and A is factorized as LL^T, where L is lower triangular.

Constraint: **uplo** $=$ **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **kd** – Integer *Input*

On entry: k, the number of super-diagonals or sub-diagonals of the matrix A.

Constraint: **kd** ≥ 0.

5: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

6: **ab**$[dim]$ – const double *Input*

Note: the dimension, dim, of the array **ab** must be at least $\max(1, \textbf{pdab} \times \textbf{n})$.

If **order** $=$ **Nag_ColMajor**, the (i, j)th element of the matrix is stored in **ab**$[(j-1) \times \textbf{pdab} + i - 1]$ and if **order** $=$ **Nag_RowMajor**, the (i, j)th element of the matrix is stored in **ab**$[(i-1) \times \textbf{pdab} + j - 1]$.

On entry: the n by n original symmetric band matrix A as supplied to nag_dpbtrf (f07hdc).

7: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **ab**.

Constraint: **pdab** $\geq$ **kd** $+ 1$.

8: **afb**$[dim]$ – const double *Input*

Note: the dimension, dim, of the array **afb** must be at least $\max(1, \textbf{pdafb} \times \textbf{n})$.

On entry: the Cholesky factor of A, as returned by nag_dpbtrf (f07hdc).

9: **pdafb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **afb**.

Constraint: **pdafb** $\geq$ **kd** $+ 1$.

10: **b**$[dim]$ – const double *Input*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \textbf{pdb} \times \textbf{nrhs})$ when **order** $=$ **Nag_ColMajor** and at least $\max(1, \textbf{pdb} \times \textbf{n})$ when **order** $=$ **Nag_RowMajor**.

If **order** $=$ **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in **b**$[(j-1) \times \textbf{pdb} + i - 1]$ and if **order** $=$ **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in **b**$[(i-1) \times \textbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

11: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \mathbf{nrhs})$.

12: **x**[dim] – double *Input/Output*

Note: the dimension, dim, of the array **x** must be at least $\max(1, \mathbf{pdx} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdx} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix X is stored in $\mathbf{x}[(j-1) \times \mathbf{pdx} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix X is stored in $\mathbf{x}[(i-1) \times \mathbf{pdx} + j - 1]$.

On entry: the n by r solution matrix X, as returned by nag_dpbtrs (f07hec).

On exit: the improved solution matrix X.

13: **pdx** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **x**.

Constraints:

if **order** = **Nag_ColMajor**, **pdx** $\geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, **pdx** $\geq \max(1, \mathbf{nrhs})$.

14: **ferr**[dim] – double *Output*

Note: the dimension, dim, of the array **ferr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: **ferr**$[j-1]$ contains an estimated error bound for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

15: **berr**[dim] – double *Output*

Note: the dimension, dim, of the array **berr** must be at least $\max(1, \mathbf{nrhs})$.

On exit: **berr**$[j-1]$ contains the component-wise backward error bound β for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

16: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **kd** = ⟨*value*⟩.
Constraint: **kd** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pdab** = ⟨*value*⟩.
Constraint: **pdab** > 0.

On entry, **pdafb** = ⟨*value*⟩.
Constraint: **pdafb** > 0.

On entry, **pdb** = $\langle value \rangle$.
Constraint: **pdb** > 0.

On entry, **pdx** = $\langle value \rangle$.
Constraint: **pdx** > 0.

NE_INT_2

On entry, **pdab** = $\langle value \rangle$, **kd** = $\langle value \rangle$.
Constraint: **pdab** $\geq$ **kd** $+ 1$.

On entry, **pdafb** = $\langle value \rangle$, **kd** = $\langle value \rangle$.
Constraint: **pdafb** $\geq$ **kd** $+ 1$.

On entry, **pdb** = $\langle value \rangle$, **n** = $\langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

On entry, **pdx** = $\langle value \rangle$, **n** = $\langle value \rangle$.
Constraint: **pdx** $\geq \max(1, \mathbf{n})$.

On entry, **pdx** = $\langle value \rangle$, **nrhs** = $\langle value \rangle$.
Constraint: **pdx** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

For each right-hand side, computation of the backward error involves a minimum of $8nk$ floating-point operations. Each step of iterative refinement involves an additional $12nk$ operations. This assumes $n \gg k$. At most 5 steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $4nk$ operations.

The complex analogue of this function is nag_zpbrfs (f07hvc).

9 Example

To solve the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} 5.49 & 2.68 & 0.00 & 0.00 \\ 2.68 & 5.63 & -2.39 & 0.00 \\ 0.00 & -2.39 & 2.60 & -2.22 \\ 0.00 & 0.00 & -2.22 & 5.17 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 22.09 & 5.10 \\ 9.31 & 30.81 \\ -5.24 & -25.82 \\ 11.83 & 22.90 \end{pmatrix}.$$

Here A is symmetric and positive-definite, and is treated as a band matrix, which must first be factorized by nag_dpbtrf (f07hdc).

9.1 Program Text

```
/* nag_dpbrfs (f07hhc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  i, j, k, kd, n, nrhs, pdab, pdafb, pdb, pdx;
  Integer  ferr_len, berr_len;
  Integer  exit_status=0;
  Nag_UploType uplo_enum;
  NagError fail;
  Nag_OrderType order;

  /* Arrays */
  char    uplo[2];
  double  *ab=0, *afb=0, *b=0, *berr=0, *ferr=0, *x=0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I,J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I,J) ab[(J-1)*pdab + I - J]
#define AFB_UPPER(I,J) afb[(J-1)*pdafb + k + I - J - 1]
#define AFB_LOWER(I,J) afb[(J-1)*pdafb + I - J]
#define B(I,J) b[(J-1)*pdb + I - 1]
#define X(I,J) x[(J-1)*pdx + I - 1]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I,J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I,J) ab[(I-1)*pdab + k + J - I - 1]
#define AFB_UPPER(I,J) afb[(I-1)*pdafb + J - I]
#define AFB_LOWER(I,J) afb[(I-1)*pdafb + k + J - I - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
#define X(I,J) x[(I-1)*pdx + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07hhc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%ld%*[^\n] ", &n, &kd, &nrhs);
  pdab = kd + 1;
  pdafb = kd + 1;
#ifdef NAG_COLUMN_MAJOR
  pdb = n;
  pdx = n;
#else
```

```
  pdb = nrhs;
  pdx = nrhs;
#endif

  ferr_len = nrhs;
  berr_len = nrhs;

  /* Allocate memory */
  if ( !(berr = NAG_ALLOC(berr_len, double)) ||
       !(ferr = NAG_ALLOC(ferr_len, double)) ||
       !(ab = NAG_ALLOC((kd+1) * n, double)) ||
       !(afb = NAG_ALLOC((kd+1) * n, double)) ||
       !(b = NAG_ALLOC(n * nrhs, double)) ||
       !(x = NAG_ALLOC(n * nrhs, double)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  k = kd + 1;
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= MIN(i+kd,n); ++j)
            Vscanf("%lf", &AB_UPPER(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = MAX(1,i-kd); j <= i; ++j)
            Vscanf("%lf", &AB_LOWER(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  /* Read B from data file */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf("%lf", &B(i,j));
    }
  Vscanf("%*[^\n] ");
  /* Copy A to AF and B to X */
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= MIN(i+kd,n); ++j)
            AFB_UPPER(i,j) = AB_UPPER(i,j);
        }
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
```

```
          for (j = MAX(1,i-kd); j <= i; ++j)
            AFB_LOWER(i,j) = AB_LOWER(i,j);
        }
    }
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        X(i,j) = B(i,j);
    }
  /* Factorize A in the array AFP */
  f07hdc(order, uplo_enum, n, kd, afb, pdafb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07hdc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute solution in the array X */
  f07hec(order, uplo_enum, n, kd, nrhs, afb, pdafb, x, pdx, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07hec.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Improve solution, and compute backward errors and */
  /* estimated bounds on the forward errors */
  f07hhc(order, uplo_enum, n, kd, nrhs, ab, pdab, afb, pdafb,
         b, pdb, x, pdx, ferr, berr, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07hhc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print details of solution */

  x04cac(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, x, pdx,
         "Solution(s)", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04cac.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  Vprintf("\nBackward errors (machine-dependent)\n");
  for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", berr[j-1], j%7==0 ?"\n":" ");
  Vprintf("\nEstimated forward error bounds (machine-dependent)\n");
  for (j = 1; j <= nrhs; ++j)
    Vprintf("%11.1e%s", ferr[j-1], j%7==0 ?"\n":" ");
  Vprintf("\n");
 END:
  if (berr) NAG_FREE(berr);
  if (ferr) NAG_FREE(ferr);
  if (ab) NAG_FREE(ab);
  if (afb) NAG_FREE(afb);
  if (b) NAG_FREE(b);
  if (x) NAG_FREE(x);
  return exit_status;
}
```

9.2 Program Data

```
f07hhc Example Program Data
  4  1  2                       :Values of N, KD and NRHS
  'L'                           :Value of UPLO
  5.49
  2.68   5.63
        -2.39   2.60
```

```
              -2.22   5.17   :End of matrix A
 22.09   5.10
  9.31  30.81
 -5.24 -25.82
 11.83  22.90                :End of matrix B
```

9.3 Program Results

```
f07hhc Example Program Results

 Solution(s)
             1           2
 1      5.0000     -2.0000
 2     -2.0000      6.0000
 3     -3.0000     -1.0000
 4      1.0000      4.0000

Backward errors (machine-dependent)
    6.4e-17     6.3e-17
Estimated forward error bounds (machine-dependent)
    2.0e-14     2.9e-14
```